Gotcha Covered

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Overview

Topic: Surface Area. This lesson provides students with a hands-on experience in which they discover the formula for calculating the surface area of rectangular prisms. Given a known volume and surface area, students create and use a spreadsheet to determine the one rectangular prism that satisfies both of these conditions. Students then use email and the web to challenge other students to solve rectangular prism puzzles that they have designed.
Prerequisite: Students should be able to compute the area of a rectangle, identify the length, width, and height of a rectangular solid, and compute the volume of a rectangular solid. Students need a basic understanding of Microsoft Excel and email.

Length of Lesson

Three 50-minute lessons

Video/Technology Hardware & Software

Math Vantage #11, Containers, Surface Area and Volume
TV/VCR preferably with remote control
One computer per group of 2-3 students with Internet access, web mail, and a spreadsheet application such as Excel.

Web Applications

Have students use email to challenge students in other classes to solve their "volume/surface area" puzzle. If the teacher has a web page, student-generated problems can also be posted there.

Learning Objectives

The student will be able to:
- develop the formula for the surface area of rectangular solids (Va. SOL Math 7.9)
- calculate the surface area of rectangular solids (Va. SOL Math 7.9, 8.10)
- given a volume and surface area of a rectangular prism, determine the length, width, and height (Va. SOL Math G.12 and 13)
- use a spreadsheet to explore the effects of varying the length, width, and height (Va. SOL Computer/Technology 8.1)
- use email to exchange prism puzzles with other students (Va. SOL Computer/Technology 8.2)

Materials and Teacher Preparations

For the Teacher:
- transparency sheets and pens

For each group of four students
- 1 Little Debbie Devil Square
- a pair of scissors
- centimeter paper

- centimeter ruler
- tape
- a six-faced paper model of a rectangular solid (the lid will not be taped so that it can be filled with centimeter cubes) Patterns for 10 different rectangular solids are attached.
- 100 centimeter cubes
- pink, blue, and yellow colored pencils

Preparatory/Pre-Viewing Activities

1. SAY: Today, we are going to be investigating the concepts of volume and surface area. Hold up a Little Debbie Devil Square. Ask the class: Can someone describe this to the class? (chocolate cake on the inside with a coating of chocolate on the outside)

2. ASK: If I were to eat just the chocolate cake which word would best describe what I had eaten, the volume or the surface area? (volume)

3. ASK: How can we mathematically calculate this volume? (Multiply the length times the width times the height.)

4. ASK: If I were to eat just the chocolate coating, which word would best describe what I had eaten? (surface area) Why? (Students should indicate that it is just the outer surface or the area of the faces of the rectangular solid.)

5. SAY: Today, our goal is to be able to calculate the surface area of this Little Debbie Devil Square.

Focus for Viewing/Other Technology

SAY: In this video clip we will review what surface area is and then determine how to calculate it for any rectangular prism. Listen carefully as surface area is described.

Time Cues

To synchronize your VCR with the time cues that are included with this lesson, zero/reset your time counter at the very beginning of the program, before the introduction and titles. Time cues are expressed as "minutes:seconds;" for example, 3:15 means three minutes and fifteen seconds.

Pause vs. Stop

When using a video interactively with students, teachers need to decide when to use **PAUSE** and when to use **STOP**. **PAUSE** the video when the anticipated discussion or activity will take less than two minutes. **STOP** for longer periods. Pausing for too long at one time can cause video heads on the VCR to become clogged which may require cleaning to correct.

Viewing and/or Online Activities

1. START the video at 03:57 just after the definition of volume has been given and when the ***Math Vantage*** logo appears. **PAUSE** at 04:25, when the cone appears.

2. Focus: We know that surface area means the number of squares it takes to cover the surface of a container. How can we mathematically figure out the surface area of a three-dimensional object? (Compute the area of the two-dimensional shapes forming the sides of the container.) **REWIND** and repeat the focus question if necessary.

3. Focus: Since we are only going to be working with rectangular prisms, let's see if we can develop a formula for finding surface area. **REWIND** to 03:57 and **PAUSE** at 04:04 where the pencil is pointing out the various sides of the rectangular prism. Place a transparency sheet on the TV screen over the figure.

4. Focus: What shapes are the yellow, blue and pink sections? (rectangles) ASK: How many of each color are there? (two) Have students identify the length, width, and height of the prism. Label these

with the letters l, w, and h on the transparency sheet.

5. Refocus: How did we say we could compute the surface area of this shape? (Compute the area of the two-dimensional shapes forming the sides of the container.) Examine how to find the area of each color rectangle using w, l, and h for the measurements. Have students recognize that Yellow = w x h, Pink = h x l, and Blue = w x l. SAY: Since there are two of each colored piece the surface area would have to equal to 2 yellow faces+ 2 pink faces + 2 blue faces. ASK: Using this information, what would the formula be? (2wh + 2hl + 2wl = SA)

Post-Viewing and/or Online Activities

1. SAY: I am going to give each group of students a paper pattern for a rectangular prism and I want you to construct the prism using scissors and tape. Leave the top open so you will be able to fill the prism with cubes. (Remind students to cut only on the outer lines and use the tabs to attach the sides to one another. Suggest to students that they make one of the largest faces the base of the prism.) Once you have created your prism, by stacking centimeter cubes or using a ruler, determine and record its length, width, and height in centimeters. Then by filling the box with centimeter cubes, figure out its volume.

2. SAY: Now I want you to use the colored pencils to shade the faces of the prism. Remember that two of the opposite faces are yellow, two are pink and two are blue. Be sure that you are coloring opposite faces.

3. SAY: Now that you have completed that task find and record the area of one yellow, one pink and one blue face.

4. ASK: How can we use this information to find the surface area? (add up the area of 2 yellow faces + 2 pink faces + 2 blue faces)

5. ASK: Since we know that Yellow = w x h, Pink = h x l, and Blue = w x l can you translate 2 yellow faces + 2 pink faces + 2 blue faces into a formula for the surface area? (2wh + 2hl + 2wl = SA) Now use the length, width and height of your prism and the formula that you have developed to calculate the surface area. ASK: Did you get the same result?

6. SAY: Now trade prisms with another group and compute the surface area using both the formula and by adding up the area of 2 yellow faces+ 2 pink faces + 2 blue faces.

7. SAY: I'm going to give each group a Little Debbie cake and for your next activity you will need to find its volume and surface area. When students successfully complete the task, allow them to eat the cake.

8. SAY: With your team, you now have a puzzle to solve. I have a mystery rectangular prism with a volume of 30 cubic centimeters and a surface area of 62 square centimeters and your job is to find its dimensions. You may use any of the materials provided including centimeter cubes, centimeter paper, or formulas but the prism you decide upon must satisfy both conditions. (Solution: 3cm. by 5 cm., by 2 cm.)

<u>Computer Application</u>
Explain to students that in this segment they will be using a spreadsheet to help them solve problems similar to the puzzle. State that you have a rectangular prism with a volume of 18 cubic centimeters and surface area of 42 square centimeters, now find the prism's dimensions. Post the attached spreadsheet on an overhead transparency (or use a computer projection system to demonstrate the actual spreadsheet) and explain that students will enter the given volume in cell D2 and surface area in cell E2. Then using trial and error, enter various combinations of length, A2; width, B2; and height, C2 to have the output in column F equal 18 and in G equal 42. Have students brainstorm strategies for selecting numbers to try. (factors of the volume)

Assessment

Give each pair of students a volume and a surface area. Some possible values are V= 12 cmΔ and SA= 32 cm≈ or V= 24 cmΔ and SA = 52 cm≈. Have stu-

dents duplicate the cells of the sample spreadsheet but they must replace the cells in column D with their prism's volume and the cells in column E with their prism's surface area. By using trial and error to select values for the l, w, and h, students are to determine the dimensions of their rectangular prism that meet the given specifications. Once the dimensions have been determined ask students to print a copy of their spreadsheet. For a further challenge, have students create their own "volume/surface area" puzzles, print a copy of their spreadsheet to verify the solution, and then email and challenge another math class to solve the puzzle.

Action Plan

1. Invite a representative from a box company or plastic container company to discuss volume and surface area in design and construction decisions.

2. Invite an architect to discuss how the concepts of volume and surface area relate to floor plans and design decisions.

3. Invite a pet-store owner to discuss cage size as it applies to an animal's needs.

4. Take a field trip to the zoo or a box company.

5. Invite a gemologist to discuss crystal formations and how they relate to various polyhedra.

Extensions

Mathematics:

- Given a sheet of paper of a specified size, create an open container to maximize the volume.
- Have students investigate the difference in the surface area of the 12-can Coke carton and the 12-can Pepsi carton. The lesson plan can be found at tlc.ai.org/lessons/cokepelp.htm where further extensions are also provided.
- Use interesting real-life containers to further explore volume and surface area. The Marriott Hotel chain uses a triangular prism for their coffee cream and stirrers' container, and a trapezoidal prism for the shower cap container.

Science: Have students use a spreadsheet to explore the change in surface area when the height of a rectangular prism is varied but the length and width stay constant. Ask to hypothesize how this would relate to a child's body temperature versus an adult's body temperature. This lesson can be found at www.eddept.wa.edu.au/centoff/graphcalc/tasks/tks71

Technology:

- At the website forum.swarthmore.edu/alejandre/ a variety of other activities involving volume and surface area can be found under the two math units titled, Crystals and Polyhedra. These units were developed by Suzanne Alejandre and have students involved in calculating the surface area and volume of rectangular prisms, naming the characteristics of various polyhedra, describing polyhedra both verbally and in writing, and finally exploring the occurrence of polyhedra in crystalline structures and "buckyballs."
- Students can use a computer drawing application to draw their own nets for various rectangular prisms.

Language Arts: Have students summarize their methods in using the Excel program to determine the rectangular prisms with a given volume and given surface area.

About the Author

Donna Rosser
Donna is a mathematics teacher and MathCounts coach at Sandusky Middle School in Lynchburg, Virginia. She has been teaching for twenty-one years and because of her dual certification in mathematics and music, has taught kindergarten through twelfth grade. Donna is an avid proponent of hands-on instruction and seeks to reach all students by utilizing a variety of teaching strategies and learning modalities. Donna received her B.A. in Mathematics from Westhampton College and her Masters of Music Education degree from James Madison University. She has conducted numerous NTTI sessions for WBRA and was also one of the facilitators for the NTTI Master Teacher Training Camp held this fall.

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SPREADSHEET WITH FORMULAS: entry in cell A1 is length

length	width	height	volume	Surface Area	Volume Test	Surface Area Test
1	2	9	18	42	=A2*B2*C2	=2*A2*B2+2*A2*C2+2*B2*C2
1	3	6	18	42	=A3*B3*C3	=2*A3*B3+2*A3*C3+2*B3*C3
2	3	3	18	42	=A4*B4*C4	=2*A4*B4+2*A4*C4+2*B4*C4
			18	42	=A5*B5*C5	=2*A5*B5+2*A5*C5+2*B5*C5

ACTUAL SPREADSHEET

length	width	height	Volume	Surface Area	Volume Test	Surface Area Test
1	2	9	18	42	18	58
1	3	6	18	42	18	54
2	3	3	18	42	18	42
			18	42	0	0

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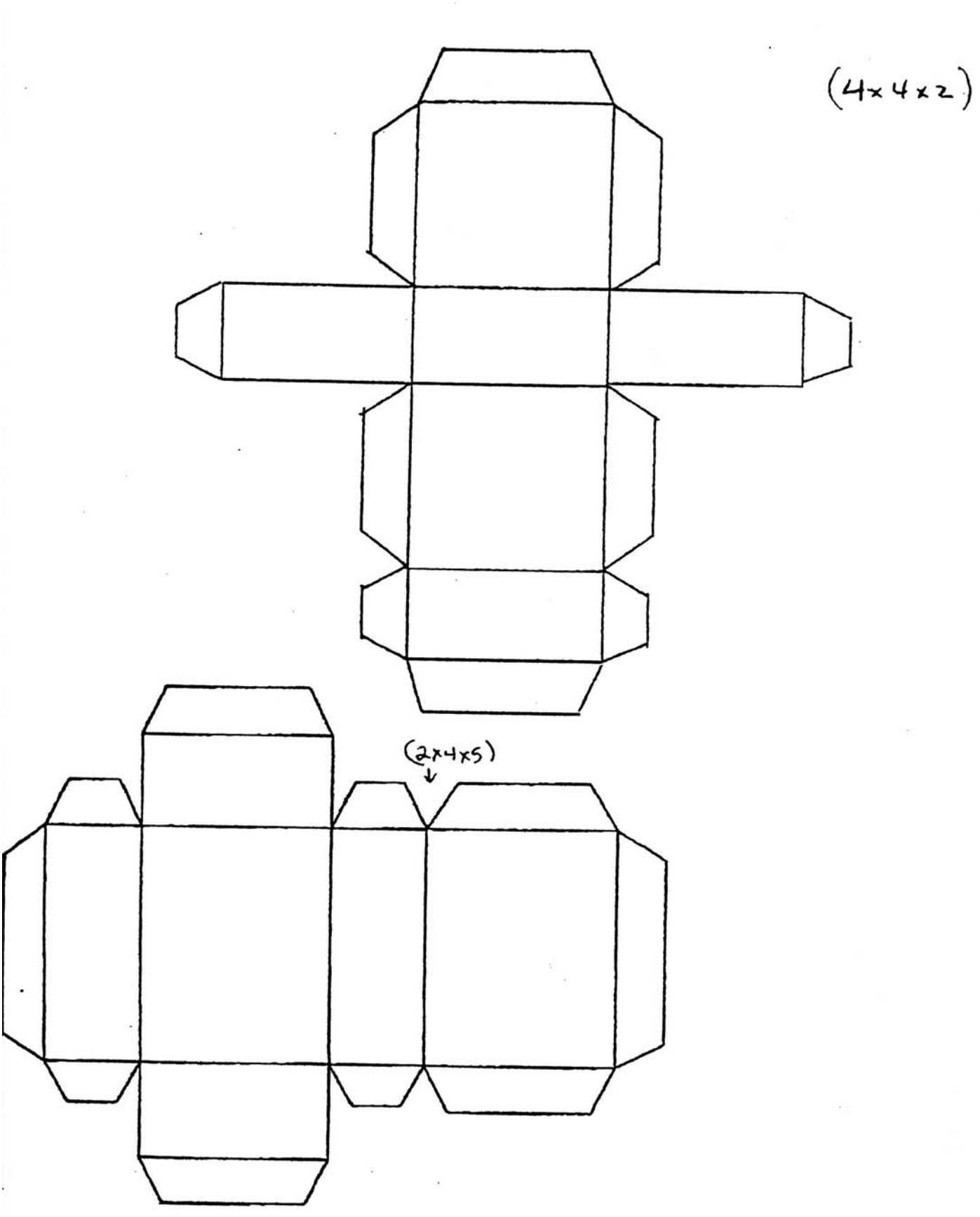

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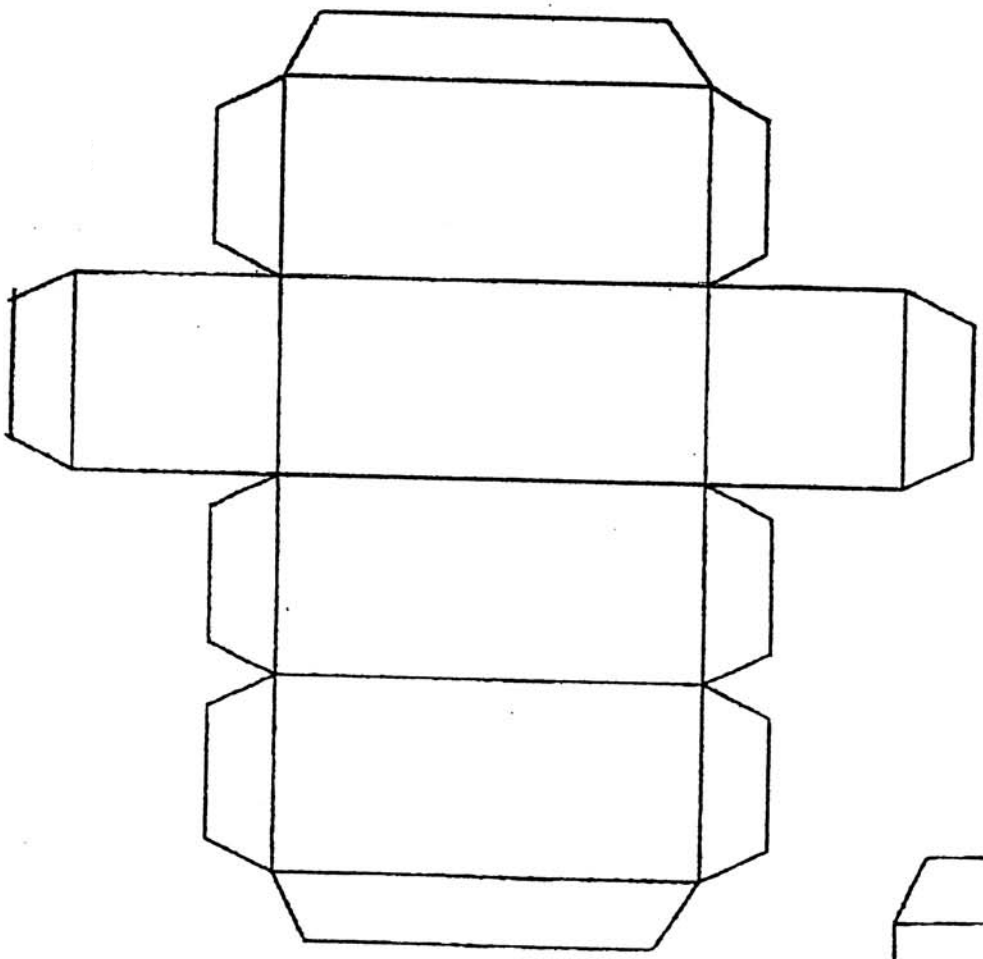

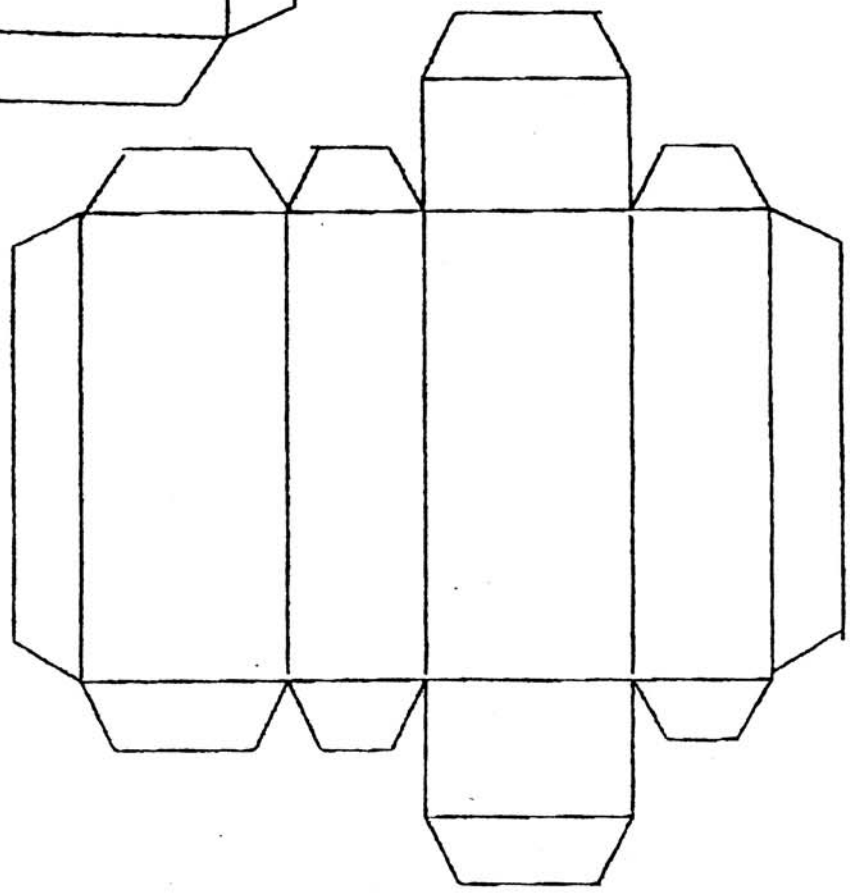

(2x3x7)

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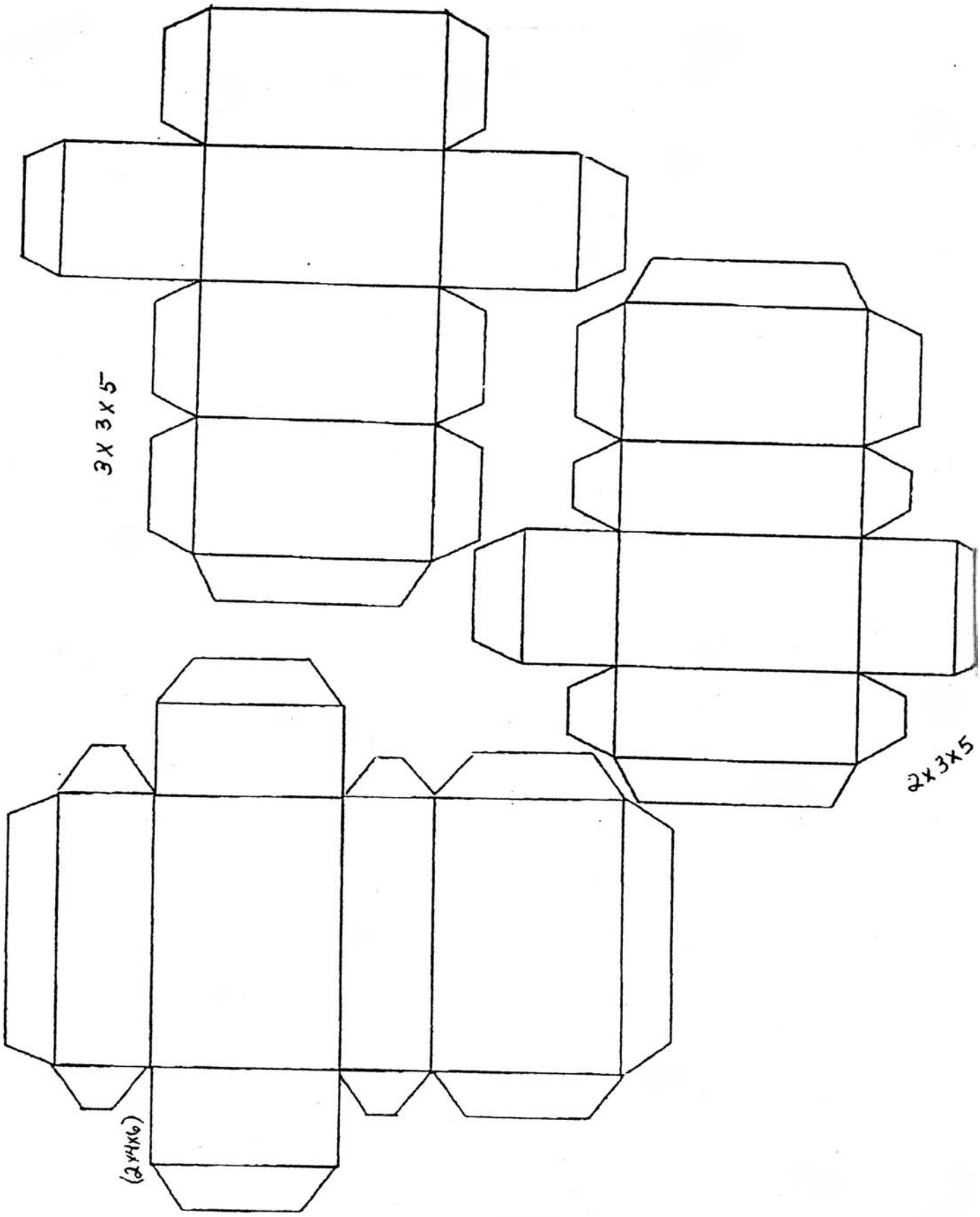

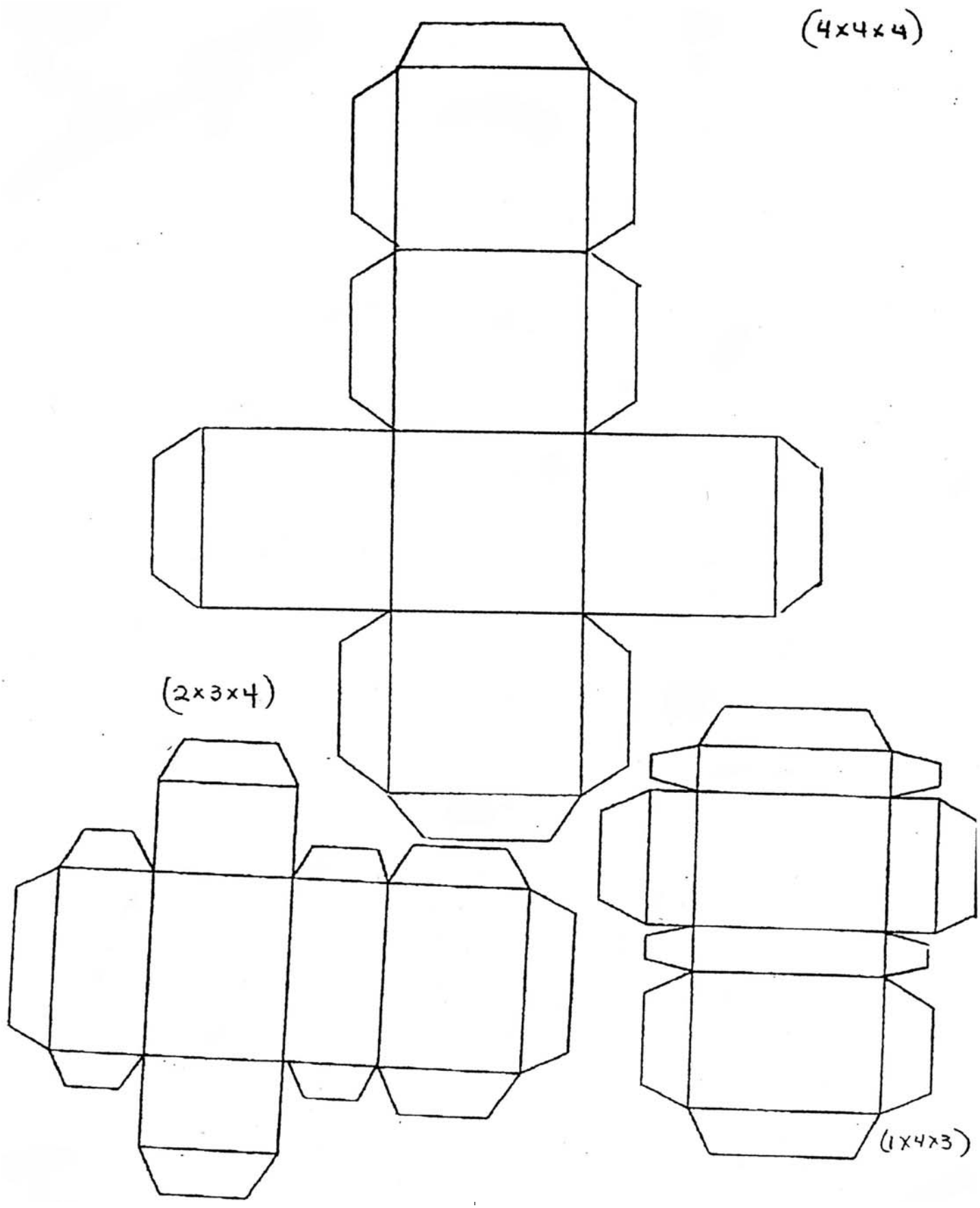
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(2×3×4)
(1×4×3)