Going Around in Circles!

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Overview

Topic: Conic Sections, Circle, Radius, Graphing. This lesson will investigate the origin of conic sections and how to derive and apply the equation for a circle. Students will explore the formation of the standard equation of a circle from both a transformational and Pythagorean approach. After an understanding of the equation is developed, the student will be asked to demonstrate how changes to the standard equation affect the appearance of the circle and use graphing utilities to justify solutions.

Length of Lesson

2 45 minute class periods

Instructional Video/Technology

Conic Sections #1, Slicing the Cone
Conic Sections #2, Circles
TI-83 Graphing Calculators
TI View Screen
Overhead Projector

Learning Objectives

The student will be able to:
- understand how conic sections are formed by having a plane slice the cone at different angles
- derive the standard equation of a circle, center (h,k) and radius r units
- justify the standard equation of a circle through a translation of the image circle
- graph circular conic equations by plotting the center and radius or by using a graphing utility
- understand some of the practical uses and applications for the properties of circles

(This lesson addresses Va. SOLs Math A.8, A.13, AII.18, AII/T.18)

(The graphing calculator will be a tool to assist in the justification of standard circles with differing centers.)

Materials

For the teacher:
- tack board
- tacks
- string (approximately 1 ft.)
- paper with Cartesian Plane (a few sheets)

For each group of 4:
- conic section cone (1 per group)
- pencils (a few per group)
- tack board (1 per group)
- tacks (a few per group)
- string (approximately 1 ft. per group)
- paper with Cartesian Plane (4 - 8 sheets per group)

For each student:
- worksheet “Post-Viewing Activity #1” (attached)

Pre-Viewing Activities

1. Tell students that they are to look at the cone-shaped object with their group. Ask them to describe what they see within the cone. Students should first make a list of shapes they see with their group. As discussion slows, have the groups share their findings with the class.

2. Write the following words on the chalkboard and have students define each.

center	radius
diameter	circle
ellipse	hyperbola
parabola	cone
coordinates	Cartesian Plane

3. Ask: What, if any, relationships exist between these words? (assessing prior knowledge)

Focus for Viewing

Say: Now that we have discussed the vocabulary on the chalkboard, we are ready to investigate a new topic. As we begin this investigation, identify and define any of the words which were not defined by the class. Also, look to see how the relationship between these words is the foundation for the concept of circles.

Time Cues

To synchronize your VCR with the time cues that are included with this lesson, zero/reset your time counter at the very beginning of the program, before the introduction and titles. Time cues are expressed as “minutes:seconds;” for example, (03:15) means three minutes and fifteen seconds.

Viewing Activities

1. Focus: Tell students that they are going to see a brief overview of circles and then discuss the information together. At (01:40) **START *Conic Section #1*** and **PAUSE** at (02:25) to check comprehension when the picture of the ellipse in space changes to a double-napped cone. Ask the class: Who discovered (developed) conic sections? Ask students for the application of conic sections given in the video or some other application which they might know. Which conic shape do you think we use the most? Justify your answer.

NOTE TO THE TEACHER
Pause vs. Stop

When using a video interactively with students, teachers need to decide when to use **PAUSE** and when to use **STOP**. **PAUSE** the video when the anticipated discussion or activity will take less than two minutes. **STOP** for longer periods. Pausing for too long at one time can cause video heads on the VCR to become clogged which may require cleaning to correct.

2. Focus: Tell students they are going to look at ***Conic Section #1*** and to watch for answers to these questions: Where do conics come from? What is a double-napped cone? What are the four conic shapes formed by the intersection of a plane and a double-napped cone? How does the relationship between the plane and the double-napped cone change to cause the different conic sections? **RESUME** the video and **PAUSE** at (03:25) to review when the introduction of the hyperbola is finished. Ask the questions again and have students discuss their answers orally.

3. Focus: Predict what the next section is going to present. Ask students: Are those the only ways a plane and a double-napped cone can intersect? Can you think of another shape created from conic sections? **RESUME** the video and **PAUSE** at (04:10) when the narrator says "indeed they are called degenerate conic sections." Ask the students: What are the other shapes formed by the intersection of a plane and a double-napped cone? What is the term used to describe this new group? Why do you think they are called degenerate conic sections?

4. Focus: Tell students that they are trying to define a circle and then apply the definition to be able to construct a circle. **RESUME** the video and **PAUSE** at (04:40) when the narrator says "equidistant from a fixed point called the center". Ask students: What is the definition of a circle? If I gave you a rope and had you construct a circle with a radius of 4 feet, how would you do it? Does your construction meet the definition of a circle? Justify your answer.

Memory Function

Most newer VCRs have a feature often referred to as “memory.” If the memory function is “on,” then when rewinding OR fast forwarding a tape, the tape will automatically stop at zero. If the counter is purposely reset or zeroed at a particular place in a program, the memory function can be used as a quick way to replay a segment or to cue to a different segment on the same tape.

5. Focus: Tell students to look for an explanation of how the terms associated with circles help to define the circle. **FAST FORWARD** to (10:55) when the double-napped cone reappears after the spacemen and **START** ***Conic Section #2.*** **PAUSE** at (11:45) when the word center (spelled centre in video) disappears from the screen to define the terms of a circle. Ask the class: What terms are essential to a circle?

6. Focus: Predict what the next section is going to present. Say: If we are going to represent a circle with an equation, what will we have to introduce to make an equation? **RESUME** the video, then **PAUSE** at (12:10) when the narrator says "calculating the distance between two points." Ask: Was our prediction correct?

7. Focus: By looking at the diagram, can you tell what theorem is going to be used to solve the problem? **RESUME** the video, then **PAUSE** at (13:40) when the equation $r^2 = x^2 + y^2$ is centered on the screen. Ask: Is there another formula like the Pythagorean Theorem? What equation has been given for the equation of a circle? Will someone write it on the board? What is the radius? Where is the center located on the Cartesian Plane?

8. Focus: Will a different center change the equation for the circle? Why? **RESUME** the video and **PAUSE** at (14:55) when the narrator completes the equation $r^2 = (x - a)^2 + (y - b)^2$. Ask: Why did the equation change? What is the new equation? What values in the equation of the circle are constant? What values in the equation of a circle are dependent upon the graph?

9. Focus: Explain to students that they are going to see another method to derive the equation for a circle. Tell students to focus on how this method is different, but attains the same final equation. **FAST FORWARD** to (15:05) when the word transformation appears on the screen and **START** ***Conic Section #2.*** **PAUSE** at (15:35) when the narrator says "form an exact replica." Ask: What is a transformation? What is the type of transformation that will be used with circles?

10. Focus: How does translation affect the ordered pair? **RESUME** the video and **PAUSE** at (16:15) when the narrator says " maps the original circle onto the image circle." Ask: If we are going to translate the center from the origin to a new ordered pair, then how will the translation affect the ordered pair?

11. Focus: Does anyone have an idea how we could represent this translation? **RESUME** ***Conic Section #2*** and **PAUSE** at (17:30) when the set of axes u and v are shown on the screen. Is it okay to introduce a new system of axes? Why would you introduce a new set of axes? Do you need to take any precautions with having a new axis?

12. Focus: How would you describe a circle on the u and v axis using x and y? **RESUME** the video, then **PAUSE** at (19:05) when several points have verified the translation. Ask: How do you compare the x-axis and y-axis to the u-axis and v-axis? Does the comparison relate to finding distance? Does the new set of axes work for all points on the circle? Discuss the difference between the signs of the translation and the standard equation. **RESUME** the video and **STOP** at (19:15) when the narrator says "the transformation has plus signs."

13. Focus: What can be a problem between the way that a translation is written and the equation of a circle?

> **Note to the Teacher**
>
> Remember that the process involved for solving for y will create both a positive and negative square root. When students are ready to enter equations, remind them that to have a complete circle, two equations must be entered.

Post-Viewing Activities

1. Compare circles using graphing utilities. Students will be given two lists of equations for circles. List 1 (activity sheet attached) has equations for circles with the same radius and different centers. List 2 (activity sheet attached) has equations for circles with the same centers and a different radius. Students must solve each circular equation for y. Enter the equations from List 1 into your graphing utilities. Have students justify the differences in the equation with the differences shown by the graphic representation. Repeat the same process for List 2.

2. Using the Cartesian plane paper, draw a circle with a fixed radius (string) and center (tack) at the origin and develop the equation for the circle. Once you have the original circle, choose other points for the center and draw the new circles maintaining the same radius. After the circles are drawn, write the translation used to create the new circles and then write the equation which represents each circle.

Assessment

1. Observe post-viewing activities.

2. Give each student a paper with circles on the Cartesian Plane and have them give the standard equation of each circle.

3. Homework assignment: Give students various equations for several circles and have them identify the radius and the center of the circle.

Action Plan

1. Have students develop a list of everyday uses of circles within their community environment. In class, students will discuss how their lives would change without each use.

2. Invite a local irrigation specialist to do a presentation for the class on how circles are used to irrigate large fields.

3. Invite a physicist to present a demonstration of physics concepts with circles.

4. Have students perform an experiment at home. Students fill a bathtub, sink, basin, etc. 3/4 full of water. Students need to find objects such as a marble, stone, golf ball, etc. to drop into the water. Students need to watch for changes that occur as different objects are dropped. (concentric circles)

Extensions

Social Sciences: Students can study the Greek mathematician, Apollonius (third century B.C.), responsible for the development of conic sections and research the time period in which he lived. What were other events which were occurring around that time? Are there any events of that time period which may have acted as a catalyst for the development of conic sections and/or circles? (related contributors are Menaechnus and Pascal)

Science: Students can do research to find scientific principles and application of the conic sections and circle. Does the development of conics have any relationship to scientific discoveries? Is the mathematician really a scientist?

Math: Student can review the method of completing the square solving quadratics in order to convert equations back to the standard equation of a circle.

Technology: Students can use search engines on the Internet to locate more information about conic sections. (search ideas: NASA Space Mathematics, Dave's Math Tables)

A publication of the 1998-99 NTTI—Virginia