Two Recipes — One Common Ingredient

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Overview

Topic: Solving systems of two linear equations in two variables using the graphing method and the algebraic elimination method. This lesson, which is a two-day plan, will first find the solutions to Systems of two linear equations in two variables using the graphing method. After a careful perusal of this method, the focus of the lesson will be on accuracy, which introduces the elimination method of solving the systems algebraically. The emphasis will be placed upon the validity of the algebraic elimination method versus the approximate validity (guesswork) of the graphing method. The students will use the graphing calculator to verify the results of their work. A final activity will be the application of these principles to practical word problems related to the number system, various mixtures, sales, and the monetary system.

Length of Lesson

2 50-minute class periods

Instructional Video/Technology

Linear Systems #3, Solving Graphically
Linear Systems #4, Methods of Elimination
TI-82 or TI-83 Graphing Calculators
Overhead Projector

Learning Objectives

The student will be able to:

- the point of intersection of the two lines belongs to both equations (Va. SOLs Math A.l, A.6)
- lines which are in the same plane do not necessarily intersect (Va. SOLs Math A.6, A.9)
- points of intersection may sometimes be found using the graphing method and "seeing" the solution (Va. SOLs Math A.6, A.9)
- some methods used for finding points of intersection are not necessarily desirable (Va. SOLs Math A.6, A.9)
- the graphing method has no accuracy if the solution is not easily "seen" and the solution becomes an approximate solution (Va. SOLs Math A.3, A.6, A.9)
- the elimination method will find the accurate solution always (Va. SOLs Math A.1, A.3, A.9)
- the key to solving by the elimination method is to produce one equation which contains one variable, then to solve (Va. SOLs Math Al, A.3, A9)
- the finding of one of the two variables creates a chain reaction which causes the other variable to become readily accessible (Va. SOLs Math A.1, A.3, A.9)
- the algebraic principles of solving linear equations with two variables can be applied to practical problems which can be stated in the form of these mathematical models (Va. SOLs Math A.l, A.2, A.3, A.9)

Materials

For the teacher:

- chalkboard and chalk
- overhead projector with transparencies & markers
- pencils or rulers

Per Student:
- 2-5 sheets of graphing paper
- straight edge
- TI-82 or TI 83 graphing calculator

Day One

Pre-Viewing Activities

1. Introduce two "lines" (pencils or rulers) to the students, being careful to emphasize that these are mere representations, since lines extend infinitely in either direction.

2. Toss the two "lines" on the floor and let the students help to discover the possible positions in which the "lines" could have fallen.

3. Conclude that there are three different things that can possibly happen if two lines are in the same plane.

4. Review the concept that a relation is any set of ordered pairs.

5. Review the use of a table of values in the process of listing ordered pairs and graphing a line.

6. Review graphing a line using the *intercept* method..

7. Introduce the *graphing* method for solving systems of two linear equations in two variables after quickly dismissing the possibility of listing ordered pairs until a solution is "seen."

8. Using the *intercept* method, graph an example of two lines intersecting at one point. This is a situation in which there is only one solution.

9. Using a graphing calculator, graph both of these lines simultaneously by employing the "yl=" and the "y2=" functions. (This will verify that there is only one solution.)

10. Using the *intercept* method, graph an example of two lines being parallel. This is a situation in which there is no solution. These lines are said to be *inconsistent.*

11. Using a graphing calculator, graph both lines simultaneously again. (This will verify that there is no solution.)

12. Using the *intercept* method, graph an example of two lines which have "different" equations but which are the same line. This is a situation in which there are infinite solutions. These lines are said to be *dependent.*

13. Using a graphing calculator, graph both lines simultaneously again. (This will verify that they coincide and therefore have infinite solutions.)

14. Using the *intercept* method, graph an example of two lines intersecting, having a solution which is not readily "seen."

Focus for Viewing

The stage having been set, the students are to look for verification of that which they have done for themselves. This will provide a nice 'closure" for their day's work.

Pause vs. Stop

When using a video interactively with students, teachers need to decide when to use **PAUSE** and when to use **STOP**. **PAUSE** the video when the anticipated discussion or activity will take less than two minutes. **STOP** for longer periods. Pausing for too long at one time can cause video heads on the VCR to become clogged which may require cleaning to correct.

Viewing Activities

1. START *Linear Systems #3*.

2. PAUSE when the narrator says "A set of ordered pairs is called a relation." Reinforce the situation by reminding the students that this set of ordered pairs exists as a direct result of using the equation of the line shown on the screen. **RESUME** viewing.

3. PAUSE when the narrator says "All we need to sketch it are these two points. And here's the line". Remind the students that two points determine a line, and therefore, only two points are necessary for graphing. **RESUME** viewing.

4. STOP when the narrator says "So calculating ordered pairs may not always work." Show the students that the futility which is shown on the screen is not efficient or acceptable. **RESUME** viewing.

5. PAUSE when the narrator says "This is how we solve a linear system graphically." Reinforce the situation by having the students recall the step-by-step approach that was used on the screen. **RESUME** viewing

6. STOP when the narrator says "so there are an infinite number of solutions for a dependent system." Review the definitions of inconsistent and dependent systems as shown on the screen and remind the students that these are special cases. **RESUME** viewing.

7. STOP when the narrator says "which means we will need another method to solve linear systems." Be sure the students understand the dilemma shown on the screen. Have them draw several examples of systems that do not have solutions which are readily "seen." **RESUME** viewing.

Post-Viewing Activities

1. Assign examples to the students for their individual solving, using the graphing method. (It is imperative that these examples have obvious solutions.)

2. As a final activity, assign some examples which do not have obvious solutions. (This will serve as a solid introduction to the activities of Day Two.)

<u>Day Two</u>

Pre-Viewing Activities

1. Begin with a brief review of the previous day's activities.

2. Recall the statement from the film ***Linear Systems #3*** which was "we will need another method to solve linear systems."

3. Graph an example of solving linear systems using the *graphing* method which does not render an accurate solution.

4. With the statement: We obviously have a serious problem here, and if we can't find a solution to it, our system of mathematics has failed us! , introduce the elimination method of solving the system.

5. Solve a system of linear equations, using the *elimination* method, in which the solution is found by either adding or subtracting. (Be careful to emphasize the basic rules of mathematics which made this possible.)

6. Use the graphing calculator to verify the solution. (This will not show the exact solution, but will show that the solution has the correct location in the overall scheme of things.) Explain once again that this method is called the *elimination* method

7. Introduce a system of two linear equations in which the direct usage off addition or subtraction does not produce the equation which is necessary for finding the solution to the system.

8. Solve this new system by using multiplication by the appropriate constants to form usable equivalent equations.

9. Give examples of using sound mathematical principles to produce accurate equivalent equations which are not usable.

10. Explain carefully the origin of the multipliers which were used to find the usable equivalent equations.

11. Introduce systems of two linear equations in which the presence of fractions is a hindrance to the solution of the problem.

12. Solve the system which contains the fractions by choosing the correct multipliers for eliminating the hindrances.

Focus for Viewing

The students are to focus on the need for accuracy and to see the advantages in using the *elimination method* for solving systems of linear equations

Viewing Activities

1. START *Linear Systems #4.*

2. PAUSE when the narrator says "We found the point of intersection was impossible to identify accurately." Recall the events of Day One and review the principles involved in the situation shown on the screen. **RESUME** viewing.

3. STOP when the narrator says 11the algebraic method used is called elimination by addition or subtraction." Rave the students work an example to reinforce their understanding of the method. **RESUME** viewing

4. STOP when the narrator says "it really doesn't matter which we eliminate, we will get the same solution." Review the principles which were involved in the method shown on the screen. **RESUME** viewing.

5. STOP when the narrator says "so the solution to this monster system of equations that we started with is x = 14/33 and 142/99". Conclude by showing the students that the elimination method ALWAYS finds an accurate solution regardless of the degree of difficulty of the system. **RESUME** viewing.

Post-Viewing Activities

1. Rave an open discussion of the weaknesses of solving by the *graphing* method versus the accuracy of solving by the *elimination* method

2. Assign problems for the students to do which involve the immediate elimination of one of the variables.

3. Assign problems for the students to do which involve the usage of equivalent equations instead of the original equations. (This will force the students to make appropriate choices of multipliers.)

Assessment

1. Have evaluative testing for the students in order to determine their levels of progress

2. Use interactive exercises to assess student accomplishment (conferences, boardwork, notebooks)

3. Determine the success of the presentation based upon the results of the testing and the interaction.

Action Plan

<u>Day One</u>

1. Assign work which involves finding "obvious" solutions.

2. Assign work which involves finding solutions which are almost "obvious." (These problems will be revisited after the activities of

3. Encourage the use of graphing calculators in order to verify the solutions.

<u>Day Two</u>

1. Assign work which involves obvious elimination techniques.

2. Assign work which involves equivalent equations and appropriate choices of multipliers.

3. Assign work which contains annoying fractions that must be eliminated by using appropriate multipliers

4. Encourage the use of graphing calculators in order to verify solutions.

Extensions

Language Arts: Have the students write an essay comparing the two methods of solving a system of two linear equations in two variables.

Science: Rave the students write the procedures for each method of solving using an "If . . . then . . ." hypothesis-conclusion approach.

Physical Education: Have the students demonstrate that the principles involved in graphing a line are much the same as a those used in correctly preparing an athletic field for play.

Special Education: Allow special education students to mark off two points and, using a string and a field marker, paint a line between the two points. If this is successful, allow them to help prepare the field for a home athletic contest.